LiP *Metamathematics in Philosophy I: Incompleteness and Intuition Dubrovnik May 26-31, 2024*

**PROGRAM** (draft v May 27)

**Sunday 05.26.24**

09:00-11:00 Incompleteness and Intuition prep seminar

*12:00-13:00* Global Instruction Group sessions [henceforth GIGS]

*prep for our evening presentations.*

*[13-15[Class the problem of intuition after Kant] zoom]*

 *15:15-18:15* Online GIGS & Student Presentations

18:30/19:00 meeting at Sesame.

Inter-University Centre LINK  [https://us06web.zoom.us/j/83054203070?pwd=nTlRdBvYKQkZOrsM6dPc7OTQGDuVpo](https://us06web.zoom.us/j/83054203070?pwd=nTlRdBvYKQkZOrsM6dPc7OTQGDuVpo.1)

**Monday 05.27.24**

09:00-11:00 *Meeting of the International Seminar Ofra Rechter via Zoom*

11:00-11:15 coffee break

*11:15-11:30 Opening Remarks* Rechter & Smokrović

11:30-12:45 *Edi Pavlovic (LMU) “Is, Ought, and Cut”.*

*12:45-16:15 Lunch break*

*16:15-17:15* Doug Blue (Pitt) Philosophical/Foundationally Interesting Work on Large Cardinals (Zoom?)

*17:30-18:45* Warren Goldfarb (Harvard) *Remarks on “Notes on Metamathematics Part I”*. (Zoom)

**Tuesday, May 28th:**

10:30-11:45 Nenad Smokrović (Rijeka) *“Does Logic Decide on the Dispute Between Realism and Anti-realism? Fitch’s Argument and its Consequences”*

*11:45-12:00 coffee*

12:00-13:15 Norbert Grazl (LMU) “Living Within One’s Means”

13:15-13:45 UIC Welcome Reception

13:45-16:00 Lunch

16:00-17:15 Michael Rathjen (Leeds): “Searching for the Ideal Framework”

**Wednesday, May 29th:**

10:30-11:45 Michael Glanzberg (Rutgers) “Logic and Logics in Natural Language”

11:45-12:00 Coffee Break

12:00-13:15 Balthasar Grabmayr: “On the Limits of Mathematics and their Philosophical Consequences” (Tübingen) Zoom.

13:15-16:15 Lunch Break

16:15 -17:30 Warren Goldfarb (Harvard) *Gödel’s General Philosophical Outlook*

17:30-17:45 break

17:45 -19:00 Assaf Kfoury (BU): “Mathematical Logic in Computer Science” Zoom

**Thursday, May 30th**:

09:00-11:00 Incompleteness and Intuition Seminar Meeting Rechter Co-Gigs: Zvonimir Šikić “Incompleteness – Easy Access: undecidability and diagonal arguments in logic”.

11:11-11:15 Break

***Student Sessions***

11:15-13:00 Saša Popović, (RijekaFaculty of Humanities and Social Sciences & Centre for Logic and Decision Theory) *From the MaxPhil Notebooks: Gödel’s Cantorian Set-Theoretical Monadology*

*Break 13:00-13:15*

*13:15-14:30 Ante Debeljuh University of Rijeka Faculty of Humanities and Social Sciences*

*Knowledge and Belief in Distributed Systems*

AFTERNOON OFF

**Friday, May 31st**

09:00-11:00 Seminar session (potentialism and modality)

11:00-11:15 Coffee Break

11:15-12:30 Volker Halbach (Oxford) *Modal Predicates* (Zoom Oxford),

12:30-13:15 A Note on Incompleteness and Intuition Ofra Rechter (Tel-Aviv)

13:15 -16:00 Lunch

**?Possibly**

16:00-18:00 **[*(tentative) Panel Session*** *on* ***learning from Bill Tait***]

*Peter Koellner (Harvard) Zoom*

*Michael Rathjen? (Leeds)? zoom*

*Doug Blue (Pitt) Zoom*

*Balthasar Grabmayr (Tübingen) Zoom*

*Ofra Rechter (Tel-Aviv)*

Abstracts

Title:**Searching for the ideal framework**

Michael Rathjen (Leeds)

**Abstract:** Proofs in mathematics often have a narrative quality to them, taking the reader on a long journey.

Sometimes the reader has to wait for new mathematical characters (like imaginary numbers) to be created so the journey can be continued. Hilbert called these novel characters ideal elements.

His conservation program was the idea that, while being important for the advancement of mathematics, ideal elements should be eliminable from proofs of concrete mathematical theorems.
Investigations by a long list of mathematicians/logicians (e.g. Weyl, Hilbert, Bernays, Lorenzen, Takeuti, Feferman, Friedman, Simpson to name a few) have shown that large swathes of ordinary mathematics can be undergirded by theories of fairly modest consistency strength.

This confirms what Hilbert surmised in his program, namely that elementary results (e.g. those expressible in the language of number theory) proved in abstract,
non-constructive mathematics can often be proved by elementary means.

The best known program for calibrating the strength of theorems from ordinary mathematics is reverse mathematics (RM). RM's scale for measuring strength is furnished by certain standard systems couched in the language of second order arithmetic. However, its language is not expressive enough to be able to talk about higher order objects, such as function spaces, directly.

Richer formal systems, in which higher order mathematical objects can be directly accounted for,
have been suggested. The price for maintaining conservativity over elementary theories, however, is that one has to use different logics for different ontological realms, allowing classical logic to reign at the level of numbers whereas higher type mathematical objects obey only intuitionistic logic. In the talk, I'd like to present some of these semi-intuitionistic systems, give a feel for carrying out mathematics within them, and relate them to systems considered in RM.

Title: **Is, Ought, Cut**

Edi Pavlovic (LMU)

**Abstract:** In this talk we offer a case study of utilizing logic to obtain philosophical results. We examine a range of deontic logics and their philosophical motivations. We distinguish syntactically between normative and descriptive statments and show, using proof-theoretic methods, that for the entire range of logics it is a (meta)theorem that the Special Hume Thesis holds, namely that no purely normative conclusion follows non-trivially from purely descriptive premises (nor, in fact, vice versa). In this way we identify a metanormative constraint which allows us to dismiss certain arguments as unconvincing based simply on their form. We finish up by laying out ways in which this project can be extended to further uses of logic in philosophy.

Title: **Living Within One’s Means**

Norbert Grazl (LMU)

**Abstract:** This talk presents a particular view on potential infinity that builds essentially on a generative process; such a process is oft assumed for different conceptions of potential infinity. However, the generative process is taken seriously and we investigate which arithmetic are justifiable from this point of view. Furthermore, as is usual in this discussion we present some correspondences to modal logics and also which kind of modal arithmetic is acceptable from this rather restricted point of view.

Title: **On the Limits of Mathematics and Their Philosophical Consequences**

Balthasar Grabmayr (Tübingen)

**Abstract**: There is a well-known gap between metamathematical theorems and their philosophical interpretations. Take Tarski's Theorem. According to its prevalent interpretation, the collection of all arithmetical truths is not arithmetically definable. However, the underlying metamathematical theorem merely establishes the arithmetical undefinability of a set of specific Gödel codes of certain artefactual entities, such as infix strings, which are true in the standard model. That is, as opposed to its philosophical reading, the metamathematical theorem is formulated (and proved) relative to a specific choice of the Gödel numbering and the notation system. Similar observations apply to Gödel and Church's theorems, which are commonly taken to impose severe limitations on what can be proved and computed using the resources of certain formalisms. The philosophical force of these limitative results heavily relies on the belief that these theorems do not depend on contingencies regarding the underlying representation choices. The main aim of this talk is to put this belief under scrutiny by exploring the extent to which we can abstract away from specific representations in the formulations and proofs of several metamathematical results.

The talk is based on technical results which are contained in the following three papers:

(2024) A Step Towards Absolute Versions of Metamathematical Results
*Journal of Philosophical Logic,*53*:*247–291[[Official Publication (open access)](https://link.springer.com/article/10.1007/s10992-023-09731-6)](2023) Self-Reference Upfront: A Study of Self-Referential Gödel Numberings (with Albert Visser)
*Review of Symbolic Logic*, 16 (2): 385–424 [[Official Publication (open access)](https://www.cambridge.org/core/journals/review-of-symbolic-logic/article/selfreference-upfront-a-study-of-selfreferential-godel-numberings/8BC653C04A7DDECC59814C64CC6D1B19)]
(2021) On the Invariance of Gödel’s Second Theorem with regard to Numberings *Review of Symbolic Logic*, 14 (1): 51-84 [[*Official Publication (open access)*](https://www.cambridge.org/core/journals/review-of-symbolic-logic/article/on-the-invariance-of-godels-second-theorem-with-regard-to-numberings/D4C5946C93D5336B10241A355B127489#article)]

Title: **On Gödel’s General Philosophical Viewpoint**

Warren Goldfarb (Harvard)

**In lieu of an Abstract: ….** The paper does talk about Gödel’s notion of mathematical intuition, arguing that it is not basic for him, but rather “epistemological optimism” underlies it (and a version of the principle of sufficient reason underlies that).

Title: **Remarks on “Notes On Metamathematics Part I”**

Warren Goldfarb (Harvard)

**Abstract:** about the pedagogical reasons for the expository and mathematical choices I made in *Notes on Metamathematics*. This presentation is intended as a framework for discussion. Questions are expected.

Title: **Mathematical Logic in Computer Science**

Assaf Kfouri (BU)

In lieu of Abstract: The talk focuses on the relationship of ***mathematical logic*** and ***computer science*** -- or, at least, ***theoretical computer science*** because there are parts of computer science today (for example, what is called ***human-computer interaction***) which are totally unrelated to mathematical logic.

Title: **Logic and Logics in Natural Language**

Michael Glanzberg (Rutgers)

**Abstract:** We live in a sea of logics, of many sorts, including sub-classical logics, sub-structural logics, extensions of classical logic, and so on. What sorts of logics can we find in natural language, and to what extent can our own languages be a guide to what the right logic is? Building on earlier work, this paper argues that natural language does not provide us with any logic directly. Even so, looking at natural language and idealizing appropriately can lead us to many logics, of many sorts. This paper goes on to argue that there are limits to what logics we can find in this way, and that some sub-structural logics will fail to emerge from a good idealization process, even though they are closely related to forms found in natural language. Hence, we can find many logics if we look at language, but there are some hard limits to what we can find.

In lieu of Title: **recent foundational/philosophically interesting work on very large cardinals**

Douglas Blue (Pittsburgh)

In Lieu of an **Abstract**: Expository Notes on some recent foundational/philosophically interesting work on very large cardinals, work inspired by that of W. W. Tait.

Title: **Modal Predicates**

Volker Halbach (Oxford)

**Abstract:** In metamathematics provability and truth are treated as properties of
formulae. When these notions are formalized in the object language via
arithmetization, they are also rendered as predicates of (codes of)
formulae. Philosophers formalizing notions such as metaphysical
necessity or knowledge usually pursue a different strategy: Necessity
and knowledge are formalized as sentential operators such as the box of
modal logic with the same grammar as the negation symbol. On the
sentential operator approach, we cannot talk about the objects that are
necessary or known within the object language. The objects, often
conceived as propositions in the guise of sets of possible worlds, live
in the metatheory. I compare the direct approach of metamathematics with
the sentential operator approach and argue that formal metaphysicians
can learn a lot form the straightforward metamathematical way of dealing
with modal notions as predicates.

Attending Graduate Student Presentations:

Title**: From the MaxPhil Notebooks: Gödel’s Cantorian Set-Theoretical Monadology**

**Saša Popović**

*University of Rijeka Faculty of Humanities and Social Sciences & Centre for Logic and Decision Theory*

— *Logic in Philosophy: Incompleteness and Intuition*, IUC Dubrovnik, 27–31 May 2024 —

**Abstract**. Kurt Gödel maintained a profound interest in Leibniz’s mathematical and philosophical writings during the greater part of his lifetime. The earliest evidence of his study of the Gerhardt edition of Leibniz’s mathematical writings comes already from 1929 (the relevant library slip is preserved in the GN folder 5/57, 050173), and most scholars agree that even after Gödel’s ‘phenomenological turn’ to Husserl in 1959, he still adhered to a more-or-less Leibnizian approach not only in logic and the foundations of mathematics but also in metaphysics (see, e.g., Van Atten & Kennedy 2003 and Rescher 2020). The recollections of Karl Menger and the reports of Hao Wang also attest to this fact, as does the great number of preserved notes and other unpublished handwritten material from Gödel’s Nachlass. Most interesting documents in this regard are the so-called MaxPhil notebooks (*Maximen Philosophie*) written between 1938 and, approximately, 1955, in which we may find an increasing number of references to Leibniz’s papers as well to the secondary literature devoted to Leibniz starting from 1943–46, especially in the MaxPhil X, XI, and XIV. The 130 pages long MaxPhil XIV written over a period of almost ten years (1946–55) may be taken as a programmatic treatise which serves as direct textual evidence of Gödel’s unrealised project of developing an *extensive metaphysical system* *based upon Leibniz’s monadology*, intertwining logic, mathematics, physics, and rational theology (cf. Crocco 2013). In this talk I wish to comparatively analyse, from a historico-philosophical point of view, Gödel’s project with the similar earlier attempt of Georg Cantor to establish a system of monadological metaphysics from the 1880s. However, this immediately turns out to be quite a challenging task due to the fact that “there is no conclusive evidence, either in his published or his unpublished work, that Gödel had read, meditated upon or drawn inspiration from Cantor’s philosophical doctrines.” (Ternullo 2018) But, upon comparing the contents of Cantor’s and Gödel’s monadology, the similarities between the two theories are quite striking: both take Leibniz as their starting point, both are informed by the relevant mathematical, i.e., set-theoretical insights, both are motivated by theological concerns, etc. Furthermore, Cantor and Gödel were both *rationalists*, they both endorsed *actual* *infinity* in mathematics, and they both subscribed to some form of *mathematical Platonism* or *realism* (mixed with hints of *idealism*). Since so many coinciding views can hardly be a product of mere chance, I will seek to uncover Cantor’s and Gödel’s respective motives and possible sources of inspiration, and to study these within wider historical, philosophical, and mathematical context.

In the first part of the talk, I will cover Gödel’s study of Leibniz’s papers, as well as his references to Leibnizian ideas in both his published and unpublished works. I will then turn to Cantor’s 1885 *Acta Mathematica* paper, underlying the philosophical and other non-mathematical aspects and motivations behind his introduction of set theory and a novel, anti-Aristotelian theory of the continuum. Finally, I will compare Cantor’s and Gödel’s attempts at developing set-theoretical monadology, showing that Gödel arrived at a theory practically identical to Cantor’s, independently of any previous insight into what Cantor published more than half a century before him.

**Selected references**

Kurt Gödel, *Collected works, Vols. I–V* (Solomon Feferman et al. eds.), Oxford University Press, 1986–2014

Kurt Gödel, *Maxims and Philosophical Remarks* (MaxPhil), Vols. IX, X, and XIV (Gabriella Crocco et al. eds.), transcriptions available online via *HAL-Science ouverte* platform (https://hal.science/).

Georg Cantor, Über verschiedene Theoreme aus der Theorie der Punktmengen in einem *n*-fach ausgedehnten stetigen Raume *Gn* “, *Acta Mathematica*, 1885.

Georg Cantor, *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (Hrsg. Ernst Zermelo), 1932.

Francisco A. Rodriguez-Consuegra (Ed.), *Kurt Godel: unpublished philosophical essays with a historico-philosophical introduction*, Basel: Birkhäuser, 1995.

Mark Van Atten & Juliette Kennedy, “On the Philosophical Development of Kurt Gödel”, *The Bulletin for Symbolic Logic*, Vol. 9, No. 4, 2003.

Solomon Feferman et al., *Kurt Gödel: Essays for His Centennial*, Cambridge: Cambridge University Press, 2010.

Gabriella Crocco & Eva-Maria Engelen (Eds.), *Kurt Gödel Philosopher-Scientist*, Aix-en-Provence: Presses universitaires de Provence, 2016.

Nicholas Rescher, “Did Leibniz Anticipate Gödel?”, *History of Philosophy Quarterly*, Vol. 37, No. 3, 2020.

Title**: Knowledge and Belief in Distributed Systems**

**Ante Debeljuh**

Faculty of Humanities and Social Sciences in Rijeka, University of Rijeka Doctoral studies, Philosophy and Contemporaneity

**Abstract.** Epistemic and doxastic logics were developed as a way of talking about knowledge and belief attribution in the most abstract of settings. Hintikka’s endeavour (1963) resulted in this domain-specific formalism reflecting the behaviour of epistemic and doxastic notions within its theoretical framework. The basis for this development was modal logic with intensional semantics, which allowed the theorists to construct the notions of knowledge and belief through relational structures. The theorists’ attempts of calibrating the formalism to fit the epistemic theory and vice versa resulted in what we now know as epistemic modelling.

Keeping this in mind, there appeared quite a few of theoretical frameworks that diverged in the way that the basic constituents of epistemic models were defined. The earlier attempts were predominantly language-based, which used structures such as sentences to play the part of the basic units of the model. I will use Stalnaker’s paper *The Problem of Logical Omniscience I* to elaborate a bit on the model he dubbed The Sentence Storage Model in order to examine some prominent issues the theorists encounter when constructing a language-based model. Some later attempts of constructing a non-linguistic model were inspired by computer sciences and developed into proper epistemic models that avoided many of the problems that the language-based models encountered. Stalnaker wrote another paper named *On the Logics of Knowledge and Belief*, in which he discusses such attempts of model building. The specific framework for modelling that I intend to discuss in my talk is called Distributed Systems Modelling (DSM), that adapts the way of talking about interconnected processors within a computer network into an externalist epistemic model.

I intend to show how we can adapt the language that epistemic logics use to talk about knowledge and belief to the DSM jargon in order to show how this way of conceptualizing epistemic situations can be a useful theoretical instrument. Furthermore, I intend to talk about the interrelationship between knowledge and belief within such structures as DSMs, as not all logics used in epistemic modelling are capable of defining them discretely. I intend to skim through some formalisms such as S4, S4.2, and S5 in order to show how this interrelationship can be defined.

**References**

Halpern, J.Y., 1987. Using reasoning about knowledge to analyze distributed systems. Annual review of computer science, 2(1), pp.37-68.

Halpern, J.Y., 1986, January. Reasoning about knowledge: An overview. In Theoretical aspects of reasoning about knowledge (pp. 1-17). Morgan Kaufmann.

Stalnaker, R., 2006. On logics of knowledge and belief. Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition, 128(1), pp.169-199.

Stalnaker, R., 1991. The problem of logical omniscience, I. Synthese, pp.425-440.

**List of Participants**: (n.b. additional participants in Panel pending confirmation).

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