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LOGIC IN PHILOSOPHY: THE CASE OF INFORMATION AND COMPUTATION

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Formal Methods and Science in Philosophy IV, Dubrovnik



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I

Introduction



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A Priori Views, or More Case Studies?

Philosophy and the Sciences

History of contacts is often much richer and
more surprising than a priori thoughts

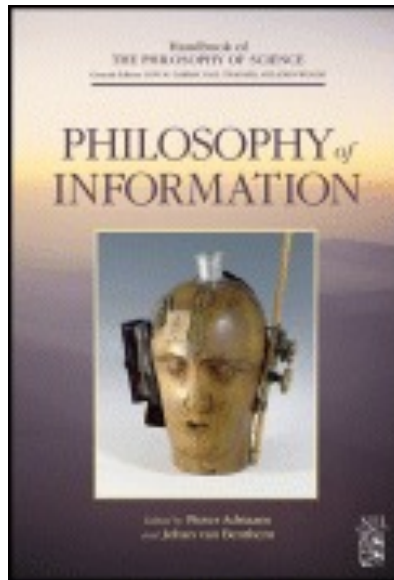
Our thread today: logic, information and computation



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Handbook of the Philosophy of Information



Long philosophical history of the notion of information

Different legitimate notions of information in the
humanities and sciences -- and even in logic



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Classical Contacts



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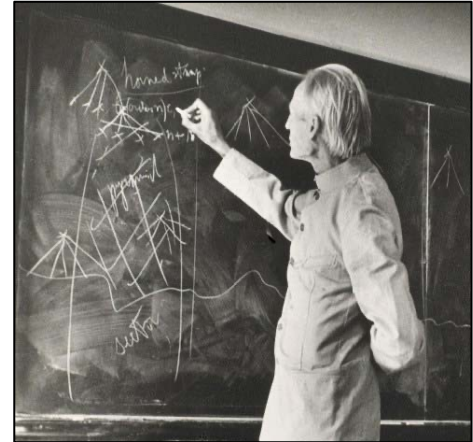
Intuitionistic Logic, Mathematics and Philosophy

Analyzing classical contacts. The ideal case study?

Constructive mathematics, **proof**

Construction and **computation**

Knowledge and information



Proof-theoretic view of meaning (Dummett, Prawitz, Martin-Löf)

Philosophical and mathematical traditions grown apart?

“State of the Union” would be useful -- but not today



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Semantic Information

Semantic Information, The Restaurant

order of 3 glasses,
new waiter brings them
two questions, one inference



information as **range of options: 6**

update through answers: **6 - 2 - 1**

questions and information seeking

final valid inference does not update:

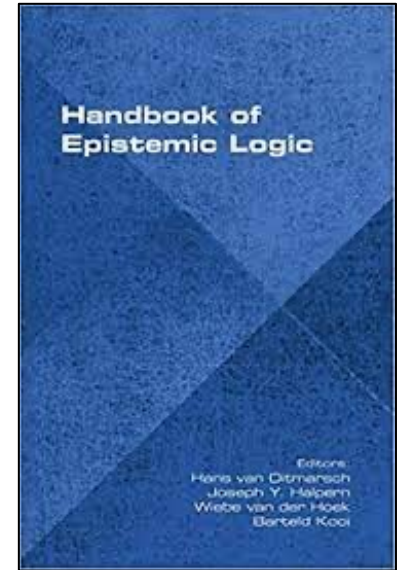
“scandal of deduction”

Epistemic Logic

common sense view of information
and knowledge

$$M, s \models K\phi$$

ϕ true in all worlds in the range of s



most scenarios **multi-agent**

knowledge about facts plus: what others know

groups as epistemic entities: common knowledge

mathematical sociology, philosophy, economics, CS



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New Lines in Philosophical Epistemology

Knowledge claims only rule out the **relevant alternatives**

Knowledge as (counterfactually) **truth-tracking beliefs**

Knowledge as **stable belief under new information**

All these analyses have an aspect of **dynamic** actions

Competing views or different aspects?



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Invariants in Science and Cognition, 19th Century



geometrical notions as invariants of **motion**

mathematical theories: structures and **transformations**

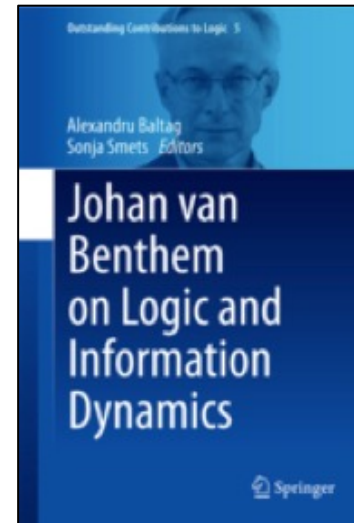
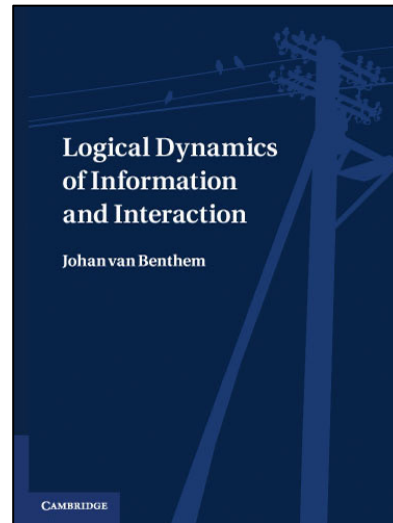
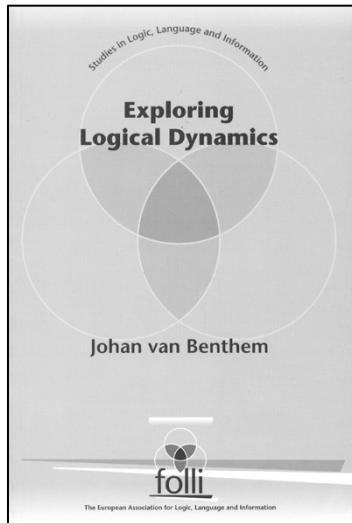
interplay of **dynamics** and statics created modern physics



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Logical Dynamics



study statics (knowledge, belief) on a par
with dynamic informational events and
determine the logical laws governing both



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IV

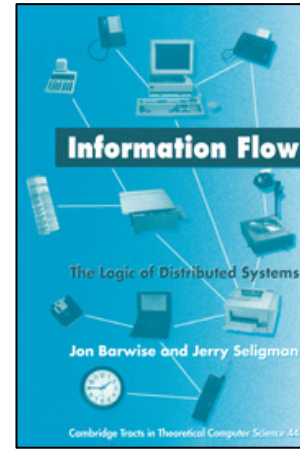
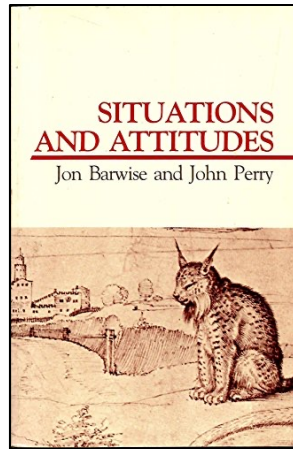
Correlational Information



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Correlational Information, Situation Theory



the universe is full of **constraints** between situations
constraints lead to **information flow** via channels

Mountain Top Seeing smoke from the **foot** of the mountain
indicates there is a fire at the **top** (long history...)



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Anti-**{Modal, Epistemic}**, But Not Really

informational dependencies in the Restaurant:

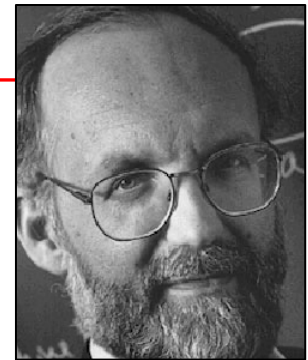
locations of the glasses constrain each other

logic of information flow across situations/models

THE JOURNAL OF SYMBOLIC LOGIC
Volume 64, Number 2. June 1999

INTERPOLATION, PRESERVATION, AND PEBBLE GAMES

JON BARWISE AND JOHAN VAN BENTHEM

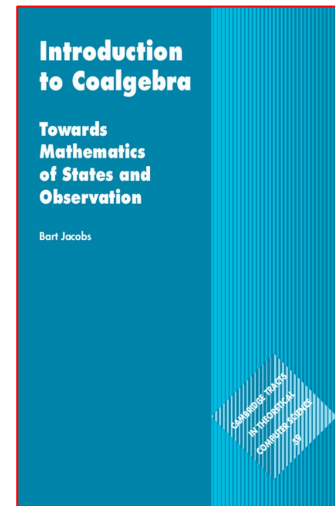
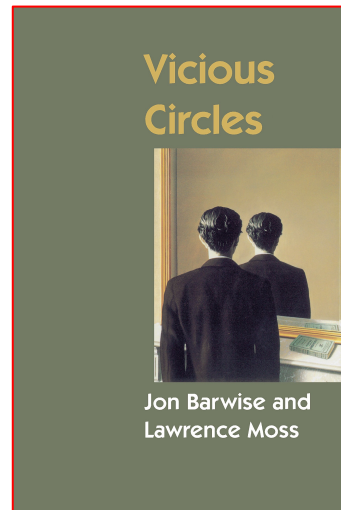




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Further ST Themes, Circularity



circularity/non-wellfoundedness among situations

science switch: precursor to modern **coalgebra**



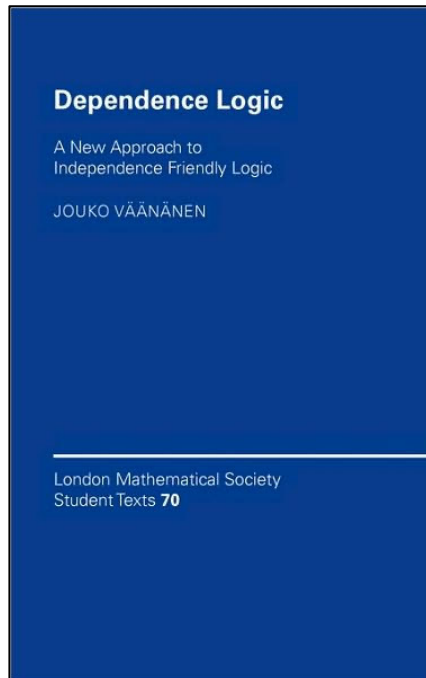
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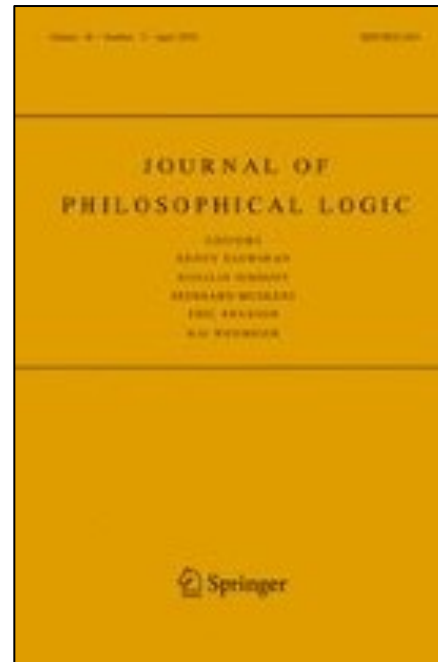
Correlation Revisited: Dependence Logics

A Simple Logic of Functional Dependence

Alexandru Baltag · Johan van Benthem



2007



2021



Modal Mini Dependence Logic LFD

team set of assignments $s: \text{VAR} \rightarrow D$ with possible **gaps**

functional dependence D_{xy} local X -values at assignment s fix the value of y : whenever $s =_x t$ in our model, then also $s =_y t$

syntax $\varphi ::= Qx \mid D_{xy} \mid \neg\varphi \mid \varphi \wedge \varphi \mid D_X\varphi$

semantics $\mathcal{M}, s \models D_X\varphi$ iff **for all t with $s =_x t$, $\mathcal{M}, t \models \varphi$**

Thm **LFD** is axiomatizable and decidable.

but if we also introduce **independence** (as in FOL)

Thm **LFD + I** is undecidable



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V

Epistemic Topology



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Empirical Inquiry

no sharp values attainable.
measurement yields ranges
approximation is crucial
limit process

crucial in sciences, also in some parts of epistemology

Margin of Error scenarios for knowledge



Can Be Modeled in Topology

open sets

outcomes of possible **observations**

approximation of values now essential

many versions of this idea:

Vickers, Parikh & Moss

also **Intuitionistic Logic!**

Topology of information states, or:

stages in temporal history of inquiry



Felix Hausdorff



Marshall Stone

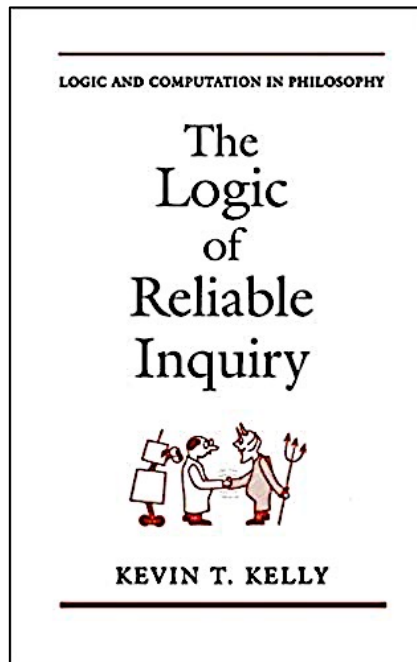


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Aside: Learning Infinite Objects

limit processes also in Formal Learning Theory





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Continuity and Learnability

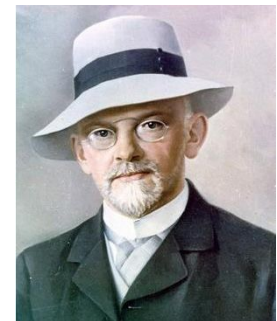
useful approximation: **continuous dependence**

knowing that measurement approaches the truth

knowing how to approximate: **uniform continuity**

independence: unlearnable parts of the universe?

“Wir müssen (nicht) wissen,
wir werden (nicht) wissen”



David Hilbert



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LCD Logic Extends LFD

The Logic of Continuous Dependence

Alexandru Baltag & Johan van Benthem

Knowability as Continuous Dependence

Alexandru Baltag, ILLC, Amsterdam

Based on joint work with Johan van Benthem.

richer modal base logics for
continuous, uniformly continuous knowledge
one more strand in growing modern interface area:



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Epistemic Topology, Interface Field

two faces

conceptual + mathematical analysis of epistemic notions

philosophy and science can work together

possible influences both ways



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VI

Computation



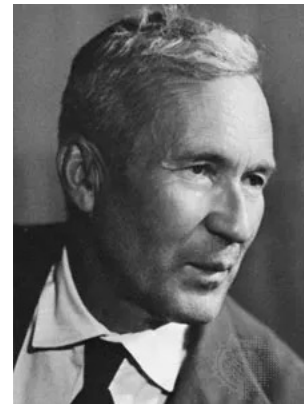
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Information as code

Concrete computation works on **code**
and so does concrete deduction

Many of the above themes need to be rethought in this setting
Turing machines, Proof theory, Kolmogorov complexity



Just One Issue: Which Information?

“Scandal of Deduction” revisited:

What information is provided by computation and deduction?

Improved access to semantic information?

Dynamics of **awareness** and **attention**?

Very live topic today

Nothing like the earlier rich semantic theory exists



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Digression: Semantic Level Still Make Sense

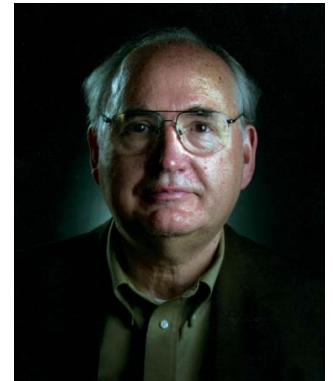
Computability in **Domain Theory**

complete lattices of 'information pieces'

Scott topology base of finite information pieces (not upset topology for inclusion)

Scott continuity ~ abstract **computability**

LFD/DCD dependence logic analysis still work here





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The Information Landscape

from **syntax** (most fine grained)

to **sets** (least fine-grained)

Many intermediate levels:

plausibility models, probability, algebra

Both directions yield insight: going **finer**, and: **coarser**

Unity of logic: interlevel connections, **translations**



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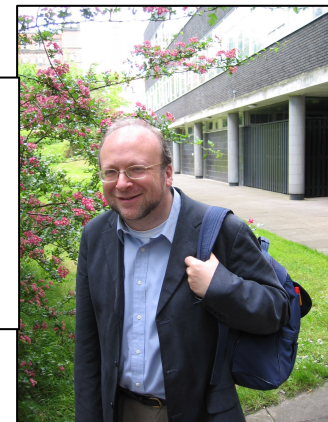
Another Deep Computational Theme: Coalgebra

Inversion of perspective: not construct objects **bottom-up**,
but **observe/analyze** outside-in

philosophical repercussions have hardly begun
e.g., for established views of meaning

Compositionality in Context

Alexandru Baltag, Johan van Benthem, and Dag Westerståhl





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VII

Conclusion

History of Ideas

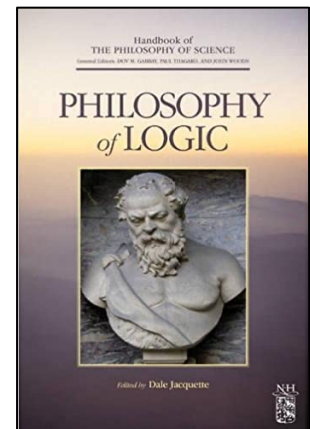
encounters of philosophy and the sciences are ubiquitous
and often not well-known

we followed just one strand:

logic, information, knowledge, computation

many more stories

need to be, and can be, told





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Coda: Philosophy and Science, Attitudes

not “fight”, but “fit”



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References

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